The Cosmological Evolution of the Average Mass Per Baryon

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(Dated: February 5, 2008)

Subsequent to the early Universe quark-hadron transition the universal baryon number is carried by nucleons: neutrons and protons. The total number of nucleons is preserved as the Universe expands, but as it cools lighter protons are favored over heavier neutrons reducing the average mass per baryon. During primordial nucleosynthesis free nucleons are transformed into bound nuclides, primarily helium, and the nuclear binding energies are radiated away, further reducing the average mass per baryon. In particular, the reduction in the average mass per baryon resulting from Big Bang Nucleosynthesis (BBN) modifies the numerical factor relating the baryon (nucleon) mass and number densities. Here the average mass per baryon, m_B, is tracked from the early Universe to the present. The result is used to relate the present ratio of baryons to photons (by number) to the present baryon mass density at a level of accuracy commensurate with that of recent cosmological data, as well as to estimate the energy released during post-BBN stellar nucleosynthesis.

I. INTRODUCTION

Big Bang Nucleosynthesis (BBN) is regulated by the competition among various two-body collisions leading to the creation of complex nuclides (mainly helium-4, with trace amounts of deuterium, helium-3, and lithium-7) built from neutrons and protons. The nucleon **number** density (n_B) is key to the BBN-predicted primordial abundances. In contrast, for studying the growth and evolution of structure in the Universe, the baryon (nucleon) **mass** density (ρ_B), including the small contribution from the accompanying electrons, plays a more direct role. Of course, the two parameters are directly related through the average mass per baryon: $\rho_B/n_B = m_B$. Given the high precision of recent cosmological data, it is important to match the adopted value of the mass per baryon to the accuracy demanded by the data when relating the baryon-to-photon ratio to the baryon mass density parameter. It is shown here that as the Universe expands and cools, m_B evolves from a value of $m_B = 939.17$ MeV at very early times, shortly after the completion of the quark-hadron transition, to a value of $m_B = 938.88$ MeV just prior to the onset of BBN at $T \approx 70$ keV, a few minutes after the expansion began. The post-BBN average mass per baryon has decreased to $m_B = 937.12$ MeV, leading to $\eta_{10} \equiv 10^{10}(n_B/n_\gamma)_0 = (273.9 \pm 0.3)\,\Omega_B h^2$, where Ω_B is the present ratio of the baryon mass density to the critical density and h is the present value of the Hubble parameter in units of 100 kms^{-1}Mpc^{-1}.

II. THE AVERAGE MASS PER BARYON

A. Pre-BBN

Prior to Big Bang Nucleosynthesis, at temperatures below the quark-hadron transition, the baryons present in the Universe are nucleons: neutrons and protons ($m_n = 939.565$ MeV, $m_p = 938.272$ MeV [1, 2]). In weak equilibrium at high temperatures there are nearly as many neutrons as protons: $n/p = \exp(-\Delta m/T) \to 1$ for $T \gg \Delta m = 1.2933$ MeV [1]. In this unachievable limit, $m_B \to (m_n + m_H)/2 = 939.17$ MeV. Note that the **hydrogen** mass ($m_H = 938.783$ MeV [2]) is used here since in the later evolution of the Universe the proton and electron go hand in hand. However, even at $T = 100$ MeV, the ratios of baryons to protons ($\equiv$ hydrogen $\equiv$ H) by number and by mass are

$$T = 100 \text{ MeV}: \quad n_B/n_H = 1 + n/p = 1.98715, \quad \rho_B/\rho_H = 1 + 1.000833(n/p) = 1.98797, \tag{1}$$

so that the average mass per baryon remains at $m_B = 939.17$ MeV. As the Universe cools further, the ratio of neutrons to protons shifts even more in favor of the lighter proton, reducing the average mass per baryon. At temperatures below ~ 1 MeV the neutron to proton ratio deviates from (exceeds) its equilibrium values, and at the onset of BBN at a temperature of ~ 70 keV, detailed calculations [3] reveal that $n/p \approx 1/7$. At this time,

$$T = 70 \text{ keV}: \quad n_B/n_H = 1 + n/p \approx 8/7 = 1.14286, \quad \rho_B/\rho_H = 1 + 1.000833(n/p) \approx 1.14298, \tag{2}$$

so that the average mass per baryon has now decreased to $m_B \approx 938.88$ MeV.

B. Post-BBN

During BBN the free neutrons and protons are rapidly transformed into light nuclides, resulting in a mixture dominated by hydrogen and helium-4, along with trace amounts of deuterium and helium-3 (any ^{3}H produced during BBN decays to ^{3}He). The mass-7 nuclides (^{7}Li, ^{7}Be) are the only others produced in astrophysically interesting abundances, but the BBN-predicted abundance of mass-7 is so small as to ensure its contribution to the analysis here is negligible (for a review of BBN and further references, see Steigman (2005) [4]). Since the binding energy of these light nuclides has been radiated away, the average mass per baryon has decreased. Until stellar processing begins, much later in the evolution of the Universe, the baryon mass and number densities are dominated by H, D, ^{3}He, and ^{4}He, and the average mass per baryon is frozen at its post-BBN value.

Introducing the abundance ratios by number with respect to hydrogen, $y_i \equiv n_i/n_H$, the post-BBN baryon number and mass densities, may be related to the corresponding number and mass densities of hydrogen,

$$n_B/n_H = 1 + 2y_2 + 3y_3 + 4y_4 + ... , \tag{3}$$

$$\rho_B/\rho_H = 1 + (m_2/m_H)y_2 + (m_3/m_H)y_3 + (m_4/m_H)y_4 + ... \tag{4}$$

Since any electrons which remain after $e^\pm$ annihilation are coupled electromagnetically to the charged nuclei, the appropriate masses to be used in eq. 4 are the *atomic* masses of the neutral atoms. The neutral atom masses adopted here from Audi *et al.* (2003) [2] are: $m_2 = 1876.12$ MeV, $m_3 = 2809.41$ MeV, $m_4 = 3728.40$ MeV.

As a zeroth approximation, ignore the contributions from D, ^{3}He (and ^{7}Li), and assume that the primordial ^{4}He abundance is $y_4 = 1/12$ (so that the primordial ^{4}He mass fraction, **defined as** $Y_P \equiv 4y_4/(1 + 4y_4)$, is $Y_P = 0.250$). This assumes that all neutrons available at BBN are incorporated into ^{4}He and corresponds to a neutron to proton ratio, evaluated just prior to the onset of BBN, $n/p = 1/7$. In this approximation, $n_B/n_H = 4/3$ and $\rho_B/\rho_H = 1.3310$, so that $m_B = 937.11$ MeV. For arbitrary Y_P (see Cyburt (2004) [5] and Serpico *et al.* (2004) [6]),

$$m_B/m_H = 1 - (1 - \frac{1}{4}(\frac{m_{He}}{m_H}))Y_P = 1 - 0.007119 Y_P, \tag{5}$$

or

$$m_B = 938.112 - 6.683(Y_P - 0.250) \text{ MeV}. \tag{6}$$

For $Y_P = 0.250$, this yields $m_B = 937.11$ MeV, in perfect agreement with the result above.

It is possible to improve upon this approximation by utilizing the BBN-predicted, primordial abundances of the light nuclides. For example, the WMAP-Only, 3-year data from Spergel *et al.* [7] correspond (see §III below) to a baryon-to-photon ratio of $\eta_{10} = 6.12^{+0.20}_{-0.25}$. Using this estimate in the fitting formulae for the BBN-predicted abundances from Kneller & Steigman [3], the primordial abundances of the light nuclides may be calculated. For $\eta_{10} = 6.12$, the BBN-predicted abundances are: $y_2 = 2.56 \times 10^{-5}$, $y_3 = 1.05 \times 10^{-5}$, and $Y_P \equiv 4y_4/(1 + 4y_4) = 0.2482$. With these abundances, $n_B/n_H = 1.3302$ (in contrast to $4/3$ in the zeroth approximation above) and $\rho_B/\rho_H = 1.3279$ (compared to 1.3310 above). However, their **ratio**, the average mass per baryon, changes from the zeroth approximation by only 0.1% to $m_B = 937.12$ MeV. It should be noted that this small difference is entirely driven by the slightly different ^{4}He abundance (0.2482 vs. 0.250) and is completely consistent with eqs. 5 & 6; in the ratio of ρ_B to n_B, the contributions from D and ^{3}He cancel to a few parts in 10^8. Thus, from shortly after the quark-hadron transition ($T \approx 100$ MeV) until stars form, reinitiating nucleosynthesis, the average mass per baryon has decreased by 2.05 MeV, from 939.17 MeV to 937.12 MeV.

C. Post-Dark Ages

During the very recent evolution of the Universe, initially small fluctuations in the mean matter density have been amplified, resulting in nonlinear perturbations which collapse under their own gravity, separating such regions from the average expansion of the Universe and leading to the formation of stars, galaxies, and cluster of galaxies. Prior to the gas being cycled through stars where hydrogen is burned to helium and beyond, the mean mass per baryon is preserved at its precollapse/post-BBN value. However, as stellar nucleosynthesis proceeds, baryons are incorporated into ever more tightly bound nuclei whose binding energy is radiated away. As a result, locally, the average mass per baryon decreases, and m_B is now distributed inhomogeneously throughout the Universe. So, while the spatially averaged value of m_B will be smaller than the post-BBN value, local values of m_B will depend on local histories of stellar nucleosynthesis. As an illustration we may adopt the recently revised solar abundances from

Asplund, Grevesse and Sauval (2005) [8] (AGS) ($X_\odot = 0.7392$, $Y_\odot = 0.2486$, $Z_\odot = 0.0122$). The ratio of baryons to hydrogen by mass is $(M_B/M_H)_\odot = X_\odot^{-1}$. For these values, $y_{4\odot} = 0.08468$ ($4y_{4\odot} = 0.3387$). Adopting the Geiss and Gloeckler (1998) [9] proto-solar D and ^{3}He abundances, $(1 + 2y_2 + 3y_3 + 4y_4)_\odot = 1.3388$. Accounting for the baryons incorporated in all the heavy elements increases the ratio of baryons to hydrogen by number to $(N_B/N_H)_\odot = 1.3554$, leading to $(m_B/m_H)_\odot = 0.9981$ and a local value of $m_{B\odot} = 936.99$ MeV. A word of caution is called for here. The AGS abundances [8] are photospheric and certainly **not** representative of the proto-solar abundances. A better choice may be the BP04+ presolar abundances adopted by Bahcall, Serenelli and Pinsonneault (2004) [10] (BSP) ($X_\odot = 0.71564$, $Y_\odot = 0.26960$, $Z_\odot = 0.01476$). In this case, $y_{4\odot} = 0.09486$ ($4y_{4\odot} = 0.37943$); since $(Z/X)_\odot$(BSP) $= 1.25(Z/X)_\odot$(AGS), the heavy element abundances (by number with respect to hydrogen) are scaled here by this factor. For BP04+, $(N_B/N_H)_\odot = 1.40021$, leading to $(m_B/m_H)_\odot = 0.99796$ and $m_{B\odot} = 936.87$ MeV. Thus, even in the chemically enriched solar vicinity, the average mass per baryon has been reduced from its value during the dark ages by only ~ 0.26 MeV.

For a BP04+ mix of abundances, along with the assumption that the helium abundance increases in proportion to the increase in metallicity ($\Delta Y \propto \Delta Z = Z$), it follows that the average mass per baryon *decreases* with increasing metallicity as $\Delta m_B \approx 0.26(Z/Z_\odot)$ MeV. This decrease in the average mass per baryon mirrors the energy released when hydrogen is burned to helium and beyond. With the simplifying assumption that stellar nucleosynthesis occurs at redshift z_*, and where Z is the large-scale average metallicity, the corresponding present day ratio of the density of the energy released to the baryon mass density is

$$\frac{\Delta\Omega_B}{\Omega_B} = \frac{\Delta m_B}{m_B}\frac{1}{(1+z_*)} \approx \frac{2.7 \times 10^{-4}(Z/Z_\odot)}{(1+z_*)}. \tag{7}$$

For the Spergel *et al.* [7] value of $\Omega_B h^2 = 0.02233$, $\Delta\Omega_B h^2 \approx 6.1 \times 10^{-6}(Z/Z_\odot)(1+z_*)^{-1}$; for $h = 0.7$, this corresponds to $\Delta\Omega_B \approx 1.2 \times 10^{-5}(Z/Z_\odot)(1+z_*)^{-1}$. It is interesting to compare this to an estimate of the observed energy density in background light. According to Fukugita and Peebles (2004) [11], the sum of the optical and far-infrared (FIR) backgrounds is $\Omega_{OPT/FIR} \approx 2.4 \times 10^{-6}$, so that (see Hauser and Dwek (2001) [12])

$$\frac{\Delta\Omega_B}{\Omega_{OPT/FIR}} \approx \frac{5(Z/Z_\odot)}{(1+z_*)}, \tag{8}$$

suggesting that stellar nucleosynthesis should occur at redshifts $1 + z_* \lesssim 5(Z/Z_\odot)$ or, that $Z/Z_\odot \gtrsim 0.2(1+z_*)$, if the observed background is taken as a lower limit to $\Delta\Omega_B$.

III. THE BARYON NUMBER AND BARYON MASS DENSITY PARAMETERS

As the Universe expands in the post-BBN era, the number of baryons in a comoving volume is conserved and the average mass per baryon is frozen at its post-BBN value until stellar nucleosynthesis begins. Since the baryon number and mass densities decrease as the Universe expands, it is convenient to express the baryon number and mass densities in terms of parameters which remain unchanged as the Universe expands. The baryon **mass** density, ρ_B, is usually written as a fraction of the *critical* mass density, defined by $\rho_c \equiv 3H_0^2/8\pi G_N$, $\Omega_B \equiv \rho_B/\rho_c$. H_0 is the present value of the Hubble expansion rate parameter, expressed as $H_0 = 100h$ kms^{-1}Mpc^{-1}. According to the Particle Data Group (PDG) [1], $\rho_c = 1.0537 \times 10^{-5} h^2$ GeVcm^{-3}. The fractional error in ρ_c/h^2, 1.5×10^{-4}, is entirely due to the uncertainty in the experimental value of Newton's gravitational constant ($G_N = 6.6742 \pm 0.0010 \times 10^{-11}$ m^{3}kg^{-1}s^{-2} [1]). This uncertainty, while not entirely negligible, is subdominant and will be ignored here. The ratio of the present baryon mass density, ρ_B, to the hydrogen mass ($m_H = 0.938783$ GeV), is related to $\Omega_B h^2$ by

$$\rho_B/m_H = 1.1224 \times 10^{-5} \ \Omega_B h^2 \ \text{cm}^{-3}. \tag{9}$$

It is conventional to compare the baryon **number** density, n_B, to the number density of the Cosmic Background Radiation (CBR) photons, n_γ. To high accuracy this ratio, $\eta_{10} \equiv 10^{10}(n_B/n_\gamma)_0$, is unchanged from the end of BBN to the present. Adopting the present CBR temperature from Mather *et al.* (1999) [13], $T_{\gamma 0} = 2.725 \pm 0.001$K $= 2.725(1 \pm 0.0004)$K, the corresponding present epoch number density of CBR photons is

$$n_{\gamma 0} = 410.50(T_{\gamma 0}/2.725\text{K})^3 \ \text{cm}^{-3} = 410.50(1 \pm 0.0011) \ \text{cm}^{-3} = 410.50 \pm 0.45 \ \text{cm}^{-3}. \tag{10}$$

Note that for the same adopted temperature the value listed in [1] (410.4), is wrong and has been corrected [14]. Combining the results above, η_{10} may be related to $\Omega_B h^2$,

$$\eta_{10}/\Omega_B h^2 = 273.42(m_B/m_H)^{-1}(2.725\text{K}/T_{\gamma 0})^3 = 273.42(1 - 0.007119Y_P)^{-1}(2.725\text{K}/T_{\gamma 0})^3. \tag{11}$$

This result is consistent with eq. 4.19 of Serpico *et al.* (2004) [6]; with the updated value of $G_{\rm N}$ [1] their coefficient 273.49 becomes 273.42. For $T_{\gamma 0} = 2.725 \pm 0.001$K,

$$\eta_{10}/\Omega_{\rm B} h^2 = 273.9 \pm 0.3 + 1.95({\rm Y_P} - 0.25). \tag{12}$$

This numerical conversion factor (for ${\rm Y_P} = 0.25$), $\eta_{10}/\Omega_{\rm B} h^2 = 273.9 \pm 0.3$, is a key result of this Letter. Note that even for the solar value of the average mass per baryon derived above using the BSP [10] abundances (BP04+), this factor increases only very slightly, from 273.9 to a local value of 274.0.

The ratio of η_{10} to $\Omega_{\rm B} h^2$ can be calculated to an accuracy of 0.1%, considerably better than the current 3 – 4% accuracy of the $\Omega_{\rm B} h^2$ determination from WMAP. However, to avoid contributing an avoidable error to η_{10} as inferred from non-BBN data, this coefficient, 273.9, should be employed when converting from $\Omega_{\rm B} h^2$ to η_{10}. For comparison, while Trotta and Hansen (2004) [15] use CAMB/RECFast for their numerical results, in their analytic results they employ a conversion coefficient closer to 275, which differs from the result here by 0.4%. The Ichikawa and Takahashi (2006) [16] conversion coefficient of 273.49 (based on Serpico *et al.* [6], using an older value of $G_{\rm N}$) is closer to the correct result, but still differs from it by more than the uncertainty of 0.3. For the WMAP-Only, 3 year data, Spergel *et al.* (2006) [7] quote (in Table 5) $\Omega_{\rm B} h^2 = 0.02233^{+0.00072}_{-0.00091}$, corresponding to $\eta_{10} = 6.116^{+0.197}_{-0.249} \approx 6.12^{+0.20}_{-0.25}$ (**not** to the value, $\eta_{10} = 6.0965 \pm 0.2055$, listed in Table 4 of reference [7] which corresponds instead to $\eta_{10}/\Omega_{\rm B} h^2 = 273.0$). It is this value, $\eta_{10} = 6.12$, which was used in §II B to calculate the primordial abundances needed for a more accurate computation of the post-BBN average mass per baryon $m_{\rm B}$.

Finally, it should be noted that the confrontation of cosmological models with the CMB data depends on *both* the baryon *number* density (through the number density of electrons) and the baryon *mass* density. As a result, such analyses depend on the numerical value of the coefficient relating the two. The codes most commonly employed to analyze the CMB data, CAMB [17] and CMBFast [18], use RECFAST [19]. The latter assumes that $m_{\rm He} = 4 m_{\rm H}$, so that $m_{\rm B} \equiv m_{\rm H}$ and, it adopts a numerical value for $m_{\rm H}$ which is too large by 0.108 MeV. In addition, the codes employ a value for Newton's constant, $G_{\rm N} = 6.67259 \times 10^{-11}$ m^{3}kg^{-1}s^{-2}, which differs slightly from the currently recommended best value [1, 14]. As a result, the conversion factor built into these codes corresponds to $\eta_{10}/\Omega_{\rm B} h^2 = 273.46$ (provided that $T_{\gamma 0} = 2.725$K is adopted), which differs from the more precise result, 273.91, by 1.5 times the error contributed by the uncertainty in $T_{\gamma 0}$. It should be noted that since Spergel *et al.* [7] use CAMB [17] (and RECFAST [19]) in their analysis, in principle their derived value of $\Omega_{\rm B} h^2 = 0.02233$ corresponds to $\eta_{10} = 6.106 \approx 6.11$. However, this small difference in η_{10} doesn't change the BBN-predicted primordial ^{4}He abundance (to four significant figures) and, as a result, has no effect on the conversion factor calculated here.

Acknowledgments

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